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Da una medaglia

THE MATHEMATICS TEACHER

Volume XXVII Number 7

Edited by William David Reeve

Algebra as a Language

By Esther D. Sweedler

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Words, words and some more words—objectives to be attained in the teaching of mathematics—courage for truth, reverence, the will to perfection—so states the report of the Standing Committee on Mathematics. One of my colleagues remarked that the committee has taken the objectives for an airplane ride. How about getting down to earth?

A very clear-cut, definite, worthy objective and one quite easy to attain is the understanding of the English language as used in Algebra. How often have you seen children represent; *A is* 6 *years older than B*, as $A=6B$; or *A is ten years less than B*, as $A=10-B$. By the second week of the term children should be impressed with the fact that they are learning a new language—the symbolic language of algebra. Just as, **Ich habe eine gute Füllfeder aber er hat eine bessere,** is translated, **I have a good pen but he has a better one,** and the English in turn may be translated back into French or German or shorthand or perhaps even a code or a cryptogram, so English is translated to algebra which in turn may be retranslated to English. (By the way, instead of the German sentence for illustration, you might add local color by using the home language of the neighborhood.) However only those who are familiar with the symbols of the new language may understand the transla-

tion. Just as the stenographer has a double job of dictation and typing so the algebra student has the job of double translation.

Once you have established the attitude of learning a new language you may adopt some of the language-teaching pedagogy. Since the algebra book does not have any vocabulary lists, you build your own from day to day and keep them on the vocabulary page of your algebra notebook. Here are but a few illustrations of what the vocabulary pages contain:

Sum: result in addition; **product:** result in multiplication; **more than:** add, plus: 8 more than x, $8+x$ or $x+8$; **subtract 8 from y:** $y-8$; **x less than y:** $y-x$; **exceeds:** is more than, 15 **exceeds** 8 by 7, 15 is 7 more than 8, $15=7+8$; **times:** multiply, a is 6 **times** b, $a=6b$; **increased by:** add, x **increased by** 5, $x+5$; **perimeter:** sum of all sides; **square root:** find the quantity which multiplied by itself will give the quantity you started with; **separate 307 into two parts:** sum of the parts is 307; **sum** (in an interest problem): investment, principal; **rate of interest:** per cent of interest; **interest:** yield, return, income; **rate:** amount of change per unit measure; **to factor:** to find the quantities which multiplied together will give the quantity you started with; **to square:** to multiply by itself; **to cancel:** to divide; **ratio:** division of one quantity by another expressed as a fraction; and any part of the verb to be, or words like has, gives, is translated by an equality sign and comparative degree of an adjective, such as **longer, older,** means **add** and is translated by plus sign.

Notice that the illustrations cover mathematical concepts as well as straightforward definitions. The beginner generally finds it simpler to grasp some of these concepts from the vocabulary point of view. How simple this problem: **If the square of a number is 36 find the number,** becomes when stated:—*What number multiplied by itself gives 36?* The concept of factoring is lost sight of very often in the numerous problems that the child factors—but suppose the teacher varies the monotony of the word **factor** by, **What do you have to multiply together to give this expression?** Unfortunately the concept **cancel** has through grade-school arithmetic meant **cross out**—thus of course when you cross out quantities nothing is left. This must be immediately corrected to the translation **divide** and the constant repetition of, $5 **divided** among 5 children gives each child $1. As a matter of fact I have attempted to make the algebra student drop the word **cancel** from his mathematical vo-

cabulary. You will likewise notice that not all the definitions are a translation from English to algebra but very many from English to English. There are innumerable reasons as to the necessity of translation from English to English. Due perhaps to the speaking of a foreign language at home, to lack of background or to a negligible amount of reading, children do not grasp the meaning, or several meanings that a word may have. In rate-distance problems, I have often found them confusing rate with time; or in interest problems I have discovered that they do not realize the word **sum** refers to the principal or investment. These vocabulary meanings can be clarified by a little dramatic pantomime and by constant repetition. The word **exceeds** has played havoc with even good students. I have had students say to me, "Well if **exceeds** means **is more than,** why doesn't the book say so?" The amount of items in the vocabulary list of course depends on the class: some classes may require extensive vocabulary lists and others practically none.

Keeping in mind that algebra is a new language I have adopted what I call the **equation sentence.** Whether you follow the method of immediately introducing two unknowns or whether you stick to the text book method of one unknown, you will find the equation sentence a tremendous help to the student.

All those who have taught elementary algebra know the difficulty that the beginning student has with a problem as simple as this:—**Three times a certain number exceeds the number by 20. Find the number.** The student becomes confused—where does the plus sign go—on which side of the equation do I put the 20? But observe how these difficulties vanish when the student is taught to set up the problem so:

Let **n** = the certain number

Three times a certain number is 20 more than the number

$$3n=20+n$$

Here he has a direct translation of the English into the algebra. I clearly recall how as a high-school student, I used to struggle with a problem of that type—first putting the 20 on one side of the equation, finding the answer incorrect in back of the book, I promptly put the 20 on the other side. Of course you could rationalize for the child. Since three times the number is heavier than the number, the 20 must be added to the right hand member in order to balance the scale. But handling the problem as a direct translation simplifies

it greatly for the beginner. Take this problem for instance: **C is now twiceas old as D. Five years later C will be 17 years older than D was 2 years ago. Find their ages now.**

Let $x =$ D's age now

$2x =$ C's age now

5 yrs. later C will be 17 yrs. older than D 2 yrs. ago

$$5+2x=17+x-2$$

Or using the method of 2 unknowns and substitution

C is twice as old as D

$C=2D$

5 yrs. later C will be 17 yrs. older than D 2 yrs. ago

$$5+ \quad C=17+D-2$$

$$5+ \quad 2D=17+D-2$$

In the above cases, the problem itself with a slight change is the equation sentence. In many problems however a word or phrase forms the sentence or it may be just implied. The power developed through the solving of problems with the equation sentence directly stated will enable the pupil to do the others with comparative ease. For instance: **A leaves 4 hours before B, but is overtaken by him in 10 hours. If A travels 5 miles an hour how fast does B travel?** Here the phrase **is overtaken** gives the equation. The pupil will set the problem up so:

	$R \times T = D$		
A	5	14	70
B	x	10	$10x$

A's distance $= B$'s distance

$70=10x.$

May I digress for a moment here? I know that there have been arguments pro and con the use of the box arrangement for the solution of problems. I have tried both methods and found that the box arrangement for problems with a relationship between the elements involved such as, $R\times T=D$ or $I=P\times R$ is much more concise and clear for the student.

This problem implies the sentence: **A ladder contains a certain number of rungs 15 inches apart. If the rungs were placed 3 inches closer together, 3 more rungs would needed. How many rungs in the ladder?**

	No. Rungs × Dist. of 1 = Height of ladder		
Before	x	15	$15x$
After	$x+3$	12	$12(x+3)$

Height of ladder before equals height of ladder after
$$15x=12(x+3).$$

In this problem the habit of looking for the equation sentence will lead the child to discover the other relationship involved.

In the following problem the word **while** gives the equation sentence: **A passenger train is going 40 miles an hour and covers 260 miles while a freight covers 210 miles. Find the speed of the freight.**

	$R\times T=D$		
Freight	x	$\frac{210}{x}$	210
Passenger	40	$\frac{260}{40}$	260

Time of freight equals time of passenger.

Some may say, "But what a waste of time rewriting the problem—the child could mentally retain the sentence that gives the equation." Not at all—I have tried it both ways and the success I have had with the sentence actually written down, on the blackboard, on the homework paper and on the exam paper, with the algebraic translation directly under each word has been gratifying. Unfortunately I have no statistics to prove it—I have not worked with controlled groups—but I know students have found this little device a great help both from their own testimony and from results. Students who had another teacher for their second semester of algebra have often come to me and said, "Gee, I can't do problems any more. My teacher doesn't use the equation sentence the way you did." I have had several pupils tell me that they showed younger brothers and sisters, who were just beginning algebra, the use of the equation sentence and they were able to do problems they could not tackle before. I will admit that I have had difficulty with students coming to me for the second semester's work—they

could see no sense in the equation sentence—a sheer waste of time writing it down—and yet they had trouble with problems. So you see the moral of this tale is—get them young, as soon as they begin algebra and then you will have no doubting Thomases.

The translation from Algebra back to English should be started at the very beginning of the term. When you are ready to discuss homework problems on the board, erase the equation sentence and then ask the pupils to read the problem from the board as it is written in their text. You will find that they like this, especially when some students open their text to check their fellow student. But that isn't the way the book says it—maybe not—but doesn't it mean the same thing? Do it in a spirit of fun and the youngsters will take to it easily. Suppose this remains on the board:

Let **n** = a certain number

$$3n-7=7+n$$

A student may translate it this way: **There is a certain number such that 3 times the number less 7 would equal 7 more than the number.** Call for other translations, then check with the text and you may find the text stating it so: If 7 is subtracted from 3 times a certain number the remainder equals the sum obtained by adding 7 to the number. Or take this blackboard problem, after you have erased the equation sentences

	$P\times R=I$		
1st investment	x	.05	$.05x$
2nd investment	y	.06	$.06y$

$$x+y=1000$$
$$.05x+.06y=56.$$

A student may translate it so: **A man had $1000, he put some in a bank paying 5 per cent and the rest in a bank paying 6 per cent. His total interest was $56. Find how much he put in each bank.** Of course the textbook states it a bit more elegantly: A man invested $1000 in two enterprises, one paying 5 per cent and the other 6 per cent. If the combined income from both is $56, how much did he invest in each?

You may vary this procedure by leaving only the algebraic sym-

bols on the board and then call for a variety of translations. For instance:

$$f=2s$$
$$t=3f$$
$$f+s+t=27$$

One student may translate so: **There are three numbers such that the first is twice the second, the third is three times the first and their sum is 27. Find the numbers.** Another may say: **Frank has twice as many marbles as Sam and Tom has three times as many as Frank. All together they have 27 marbles. How many has each?** By this time students are well aware that the number of equation sentences corresponds to the number of unknowns used.

At the close of a recent meeting of the New York Society for the Experimental Study of Education in which Mr. Nathan Lazar suggested the early introduction of two unknowns in problem solution, I heard Mr. John Swenson of Wadleigh High School say, "But everybody is doing it." So to forestall the same criticism to my very simple suggestion may I state that modes in teaching mathematics do not travel with the speed of popular songs. I believe the average teacher is rather poorly trained in the pedagogy of his particular subject. Personally, I welcome all suggestions from fellow teachers, very often I ask for them directly or obtain them by observation of their teaching procedure. I eagerly look forward to the monthly issues of *High Points* and the *Mathematics Teacher* in hope of finding those little hints which seem so obvious and yet—you know, like the invention of the rubber soap dish which netted its inventer hundreds of thousands of dollars.

Diagrams in Algebra

By Arnold Dresden
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The statement that "In geometry we prove things, in algebra we do things" represents a point of view not infrequently taken by young people who, after four years' study of mathematics in the high schools, have undertaken a period of further study in college. And woe to the teacher of college freshmen who attempts to extend to his college classes in freshman algebra the pernicious habit of persistently asking "Why?" Many of his students get hopelessly lost from the start. If they are given rules and sample problems, they can utilize their "training," follow the rules, copy the procedure from the samples and carry on indefinitely a great deal of quite meaningless and on the whole quite useless work—they can play the game, but not well enough to win. Should they be asked to "prove" that the sum of the binomial coefficients $\binom{n}{k}$ taken from $k=0$ to $k=n$ is equal to 2^n, they would not respond at all, or perhaps they would repeat parrotlike a memorized lesson; at best they might manipulate a sum like $\binom{5}{0}+\binom{5}{1}+\binom{5}{2}+\binom{5}{3}+\binom{5}{4}+\binom{5}{5}$ until they obtain 32. They are "doing" algebra, not understanding it.

It should not be necessary to argue that such a difference between the ways of looking at algebra and at geometry has a very bad effect upon the appreciation which the student gets of the character of mathematics. What can be done to overcome this obstacle to his mathematical development, which prevents his understanding the unity and logical structure of the subject? Some preliminary observations will indicate perhaps a direction in which we may look for an answer to this question.

Between the algebra as taught in most of the schools and many of the colleges (let us call this "formal algebra"), and the mathematical subject which bears this name, there exists a difference not unlike that between the "intuitive geometry" of the new dispensation and the "demonstrative geometry" of the unregenerate dark ages; but, with a distinction. Intuitive geometry derives some justi-

fication from the fact that the student finds in the precipitate of his everyday experiences some things which are readily linked up with this subject, and from the fact that these connections are actually made use of. But, for the greater part of formal algebra lines of communication with experience do not exist, except for the unusual student; and the few communications which could be utilized are allowed to deteriorate for want of use, except by the unusual teacher.

The teaching of algebra is clearly unsatisfactory. The "practical" ends which the subject is supposed to aim at are indeed badly served by it. The graduates who have "credit" in algebra may know all the formulas and ingenious devices; but they are likely to be quite indiscriminate in their use of the former and somewhat overawed by the mysteries of the latter. The teacher of college algebra testifies on every occasion that his students are deficient in elementary algebra, the instructor in trigonometry or calculus complains of the poor preparation in algebra, and so on; the instructor in course $n+1$ invariably laments the students' inadequacy in the subjects covered by courses 1 to n inclusive. Can there be any doubt that formal algebra completely and unequivocally fails to justify itself? Two directions in which an improvement of this state of affairs may be looked for suggest themselves. On the one hand, those aspects of the students' experience which have a bearing on the abstract concepts of algebra should be drawn upon fully. In the second place algebra should be treated as a deductive science. Would any one maintain that students, who can develop a logical argument in geometry, can not do so in algebra?

Both of the above suggestions require further elaboration. While it is with the second that I shall principally be concerned here, the two are so intimately connected that I can not leave the first entirely out of consideration.

The difficulty which students experience in attempting to prove theorems in algebra arises from their abstract character and from the absence of concrete interpretations that may lead to an understanding of the abstract concepts. The theorems of algebra are not more abstract than those of geometry, if the latter are correctly understood. For the theorems of geometry do not deal with rods and balls, but with lines and points, which are "things" to which no meaning is to be attached. But we rarely present geometrical

theorems to young people in this abstract form—we draw diagrams to aid their understanding and to guide their reasoning. Most of us do the same thing in the study of more advanced parts of geometry.

Can we not "draw diagrams" in algebra? We do draw diagrams sometimes when we introduce graphical interpretations of algebraic formulas. The writings of Greek mathematicians give us many examples of such use of diagrams (see, e.g., Heath, *Manual of Greek Mathematics*). These interpretations are doubtless useful in breaking down the barriers between algebra and geometry. But their possibilities are rather limited; they make possible a geometric analogue of the algebraic theorem we wish to prove, rather than a proof of such theorem. The value of such a procedure may well be doubted if we are interested in developing algebraic skill, because it dodges the algebraic methods altogether.

By "diagrams in algebra" I mean something quite different. What we need to carry over into algebra is not the pictorial aspect of the diagram, but rather its relation to the theorem it assists us in understanding. This relation is simply that of a special case to the general case. For when, in connection with the theorem that the midpoints of the sides of a quadrilateral are the vertices of a parallelogram, we draw the familiar diagram, we substitute for the points and lines implied in the statement of the theorem certain collections of tannic acid, or of graphite or chalk. These are special instances of material objects for which the postulates underlying our geometric structure are sufficiently nearly satisfied to keep us out of difficulties. It is this quality of the "diagram" which can be carried over immediately into the field of algebra. By a "diagram" in algebra, we mean nothing but a special case which can be used to clarify the content of a theorem, or the meaning of a definition, or the real significance of an argument. Just as the diagram in geometry frequently suggests a method for attacking a problem, so the "diagram" in algebra has value in indicating the sources of the generalizations that are being dealt with.

Some judgment is required if the most effective special cases are to be selected. In the proof for the expansion of $(a+b)^n$, for positive integral exponents n, it would not be particularly effective to take as a "diagram" the special case $(2+5)^n$; but it would be very useful to consider $(a+b)^3$ and $(a+b)^5$. A student who has learned

in the right way that the product of $a+b$ by $a-b$ is equal to a^2-b^2, should not require pencil and paper to multiply 52 by 48. At a later stage the student may meet the theorem that a superior limit for the positive real roots of the algebraic equation $a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n=0$ is given by $1+\sqrt[k]{G/a_0}$, in which k and G have definitely specified meanings. The mere statement of such a theorem is frequently puzzling to a beginner. The person who has become accustomed to the use of "diagrams in algebra" will take a numerical special case in which n, a_0, a_1, $\cdots$, a_n are replaced by integers. This will aid in bringing about a sense of familiarity which is expressed by saying that we know "what the theorem is about."

It is clear that in the early stages of mathematical training purely numerical instances will provide the most useful "diagrams," perhaps the only useful ones. With the growth of understanding and of familiarity, the new concepts of last week can be used as special cases for the newer concepts of to-day. When the *Remainder Theorem* is reached, a cubic polynomial with general coefficients may be an effective diagram, whereas at an earlier stage it may have been quite useless; in the study of arithmetic progressions of higher order, the general progression of the first order may serve as a significant example. The systematic use of "diagrams" brings about constant and cumulative reviewing in a perfectly natural manner. But, what is more important is that by aiding in the clarification of new concepts, it unifies the content of algebra, it prevents a barrier from growing up between algebra and geometry, it facilitates the treatment of algebra as a deductive science. Thus it becomes a valuable aid in significant mathematical education.

Notice to Subscribers

"Locus Makes a Plea"

By D. McLeod
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To the average pupil—or, for that matter, to the brighter one—the topic called *Locus* in any geometry text is generally distasteful. The reason may be far-reaching. Perhaps the subject is put in there because it has always been an integral part of an approved geometry course. Tradition! How many things are done in thy name! To many *Locus* is a detached section having no place in the unity of the whole. In some cases, it may be, the treatment of the topic is so superficial that genuine interest is impossible. Again it may well be that its Latin name marks it off as a thing of mystery. Whatever it is, the fact remains that, in many cases, the pupil can "get by" what he regards as a necessary evil only by sheer memorizing of subject matter which to him can have no practical application. And so *Locus* teaching goes on and on and such has been its history.

The conscientious teacher realizes this ever-present difficulty and asks: "Why should these things be? If *Locus* is worth studying it is worth studying well. If it is useless, then why include it at all? A mere smattering leads only to distaste. Has Locus really a place and value in a well-rounded geometry course?"

The answer of the writer to the past question is a decided "yes." The conclusion has not been reached without careful thought. Constant readers of The Mathematics Teacher are seriously concerned about warning notes re the decline of the importance of geometry in the present school and college curricula. The forces must be rallied and a good front presented against the ever-increasing criticism of our mathematics program. As a chain is just as strong as its weakest link so the parts of geometry most open to criticism should be strengthened by having their value made plain. In case *Locus* may be further criticized let it be stated here and now that it is a topic of which a great deal can be made—in short, that it has both a significance and a value.

The generally accepted definition of *Locus* is "the path traced by a point moving in obedience to a law." The very statement of the definition gives something novel and provides a digression

from the ordinary routine. We are in the region of discovery.

Poor pupils have found it difficult to remember the meaning of the term under discussion. The definition really embraces three things. (1) A point moves. (2) That point moves in obedience to a law. (3) A path is thus traced. The easiest example of *Locus* is the circumference of a circle. Surely no pupil can fail to grasp (1) that the moving chalk-arm of a compass obeys a law and (2) that the path or line traced is not one drawn at random but a definite, well-defined, easily recognizable line which has been given a Latin name just for convenience. If the word *Locus* is confusing the teacher might substitute "the path traced by." The class, for example, might, at the beginning be asked questions such as the following: (1) What is the path traced by a point which moves so that it is always the same distance from a fixed point? (2) What is the path traced by the door-knob as the door is being opened? Later, the word *Locus* will fit into its proper place.

In regard to the common difficulty of remembering the "form" which the required locus will take, it might be suggested that the pupils be told that, as far as their work is concerned, there are but two forms of locus, viz.:—the circle and the straight line. (Note that here "circle" means the circumference of the circle or part of same.) Further these two forms have something in common. They are both *even* in their course. There is about each a uniformity which is at once apparent. Incidentally, the so-called "morality" of geometry can be shown at this point. Can a crooked line be a locus? Does a crooked individual obey the law?

The subject-matter of locus is generally as follows:

(1) The right-bisector of a given straight line is the locus of a point which moves so that it is always equidistant from the ends of the line.

(2) To find a point equidistant from three given points not in the same straight line.

(3) The bisector of the angle formed by two straight lines is the locus of a point which moves so that it is always equidistant from the two given lines.

(4) To find a point equidistant from three given straight lines.

Numbers 1 and 3 are theorems. Numbers 2, 4 are corresponding problems.

In the actual learning of the above propositions, numbers 1 and 3 present especial difficulty. At the outset a pupil does not under-

stand why in number 1, for example, the proof should have two parts—why, in other words, the first part is not sufficient without having to proceed to prove the converse of it. To get over this difficulty it might be a good idea to condense the enunciation of proposition 1 thus: The right bisector of a given line contains all points equidistant from the ends of the line. Likewise with proposition 3. Now, how is one to proceed from there? The class might be told that all points comprise (a) any point on the right bisector as drawn and also (b) any point anywhere else. Now the reason for the two parts of the proof will be clear. First we must show that any point on the right bisector actually drawn is equidistant from the ends of the line and, second, we must prove that any point whatsoever which is equidistant from the ends is really on the right bisector.

What is the value and significance of *Locus?* This possibly is the chief part in our plea for the topic.

1. It is clear that nowhere, possibly, in the study of geometry does one find such certainties or such unqualified statements as are found in locus study. In some other departments of his school work the pupil may fill in a blank in either of two ways and be correct each time, but in an exercise on locus one thing and no other will be correct. The sure ground on which he stands gives him a confidence and a resulting interest which are far-reaching.

2. Here we have learning only by actual doing. Pupils will want to trace possible positions of a moving point for themselves and so actually derive the correct locus. Surely this is a valuable educational practice.

3. In the study of locus problems we build better than we know. The foundations are laid for more advanced work in graphs or even in the calculus.

Exercises like the following are typical in the treatment of the topic.

1. To find a point inside a quadrilateral equidistant from three sides.

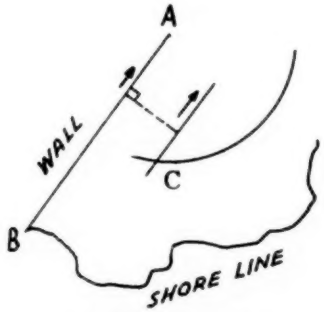
2. A point moves at a fixed distance from a given line. What is its locus?

3. Locate a point equidistant from two given intersecting lines and a definite distance from their point of intersection.

4. Find the position of a point which is a fixed distance from a given line and also a fixed distance from a given point.

5. What is the path traced by the axle of a wheel of a car as the car moves? An illustrative exercise may here be given. To find a point 250 feet from a straight breakwater wall and also 500 feet from a lighthouse at the end of the wall. What

line (or locus) will contain *all* points 2 inches (scale $1'' = 250'$) from A? A circle with A as center and a radius of $2''$. The required point is on this line and nowhere else. Note the *certainty*. What line has all points on it which are exactly $1''$ from AB?



C is the required point

A line $\parallel$ *AB* and $1''$ from it in a perpendicular line. What is true about the point of intersection of the two loci? Where then must the required point lie? What opportunity for neatness of diagram, reasoning, broad outlook, in this exercise!

In conclusion it may be stated that we hope our plea is worthy. A topic which can be used so effectively to prepare our pupils for better work in the other sections of geometry cannot be treated off-hand.

"Drink deep or drink not from the Pierian Spring."

WALTER PITKIN, speaking to a group of students at Columbia University recently said: "Mathematics has reached out beyond language in its use of logical symbols, We have learned that symbols can mean anything. Mathematical symbolism is more nearly perfect than any other."

The Mayan Calendar

By John Kinsella
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Professor Judd of the University of Chicago in the April 1929 "Mathematics Teacher" while urging teachers of mathematics to keep in mind the informational values as well as the computational uses of mathematics made the following interesting statement: "Number is a general system. It was invented by the race as a useful system by means of which anyone who is in command of the number series can arrange all kinds of miscellaneous experiences and thus greatly facilitate intelligent thought about particular items." It seems to be a general theorem that races in their evolution toward a higher degree of civilization have almost inevitably developed a number system for the expression of order and precision. This theorem has a striking application on our own continent. Centuries ago a people called the Mayas occupied an area of over 50,000 square miles spread over Yucatan, southern Mexico, Guatemala and Honduras. Despite the fact that this race was entirely isolated from contact with European civilization, until the arrival of the Spaniards about 1517, it had advanced to a high degree of culture. An outstanding achievement of the Mayas was the development of a very exact and rather intricate calendar. Spinden, a Mayan scholar, pays tribute to the originator of the Mayan calendar in these glowing phrases: "He (the early Mayan) carried out observations of an astronomical nature and created the very tools of thought with which to work. He developed a permutation which gave fleeting units of time a personality and a name; he evolved a hieroglyphic method of recording essential facts in relation to each other; he devised place value notation for number which made possible the first arithmetic. More than all this, he so impressed himself upon his fellow men that the highly intellectual machine which his mind set in motion continued to function without a fault until it was wrecked upon the burning scaffold of the Inquisition. The Mayan calendar which this man invented ran without the loss of a day for 2148 years and controlled the religious

and civil life of several nations." It is the purpose of this paper to consider that calendar.

The Mayan year (Haab) was divided into 18 "months" of 20 days with a closing period of 5 days called the 5 unlucky days. These "months" were called uinals. These uinals in order of occurrence were Pop, Uo, Zip, Zotz, Tzec, Xul, Yaxkin, Mol, Chen, Yax, Zac, Ceh, Mac, Kankin, Muan, Pax, Kayab, and Cumhu. In numbering the days of these months the Mayans used a plan peculiarly their own. The first day of the Mayan month was numbered 0 instead of 1. For instance, 0 Pop would correspond to our January 1. They consistently used the idea of elapsed time in contrast with our inconsistent use of time units. We write July 16, 1933, 2:00 P.M. The "2:00 P.M." means that two hours have elapsed since noon while the "July," the "16," and the "1933" signify time intervals that have not passed. Spinden summarizes this point in admirable fashion: "To the Mayan mind zero was not nothingness but completion, and it seems that the people may have had a truer philosophy of number than we ourselves can boast. They registered only elapsed or completed units of time, while we make an illogical use of current units in all our larger measures."

It will be noticed that the portion of the calendar so far described is based upon the astronomical unit, the year. The remaining part of their calendar had a more arbitrary basis. The days of the month were numbered in the manner exemplified by "O Pop," or "7 Pop" or "8 Kan" or "15 Men." Besides this numeration the Mayans had 20 day names instead of 7 as we have. They were Imix, Ik, Akbal, Kan, Chicchan, Cimix, Manik, Lamat, Muluc, Oc, Chuen, Eb, Ben, Ix, Men, Cib, Caban, Eznab, Cauac and Ahau. Prefixed to each of these day names was a number ranging from 1 to 13. To show how the Mayans used these 13 numbers in co-ordination with the day names we will arbitrarily attach a number to one of the day names and arrange the others that follow. Suppose "5 Imix" represented a certain day. Then the days that followed would be 6 Ik, 7 Akbal, 8 Kan, 9 Chicchan, 10 Cimix, 11 Manik, 12 Lamat, 13 Maluc, 1 Oc, 2 Chuen, 3 Eb, 4 Ben, 5 Ix, 6 Men, 7 Cib, 8 Caban, 9 Eznab, 10 Cauac, 11 Ahau, 12 Imix, 13 Ik etc. The series of 13 numbers and 20 day names kept repeating each other in the cyclic order as shown. In our system a day's name is repeated every 7 days, in theirs 20 days. Due to the variation of the number of days in our months it is not easy to calculate

when a day name and its number, e.g. "Wednesday the 6th" of a certain month, will be repeated. In the Mayan scheme, since the least common multiple of 13 and 20 is 260, it is certain that a day name with its accompanying number, e.g. 7 Akbal, will occur again in exactly 260 days (tzolkin).

Up to now you have probably observed we have mentioned 4 co-ordinates in locating a specific day—the day numbers in the "1–13" series, the day names of which there are 20, the numbers of the days of the month of which there are 20, and the month names of which there are 18. In writing the first day of the year our ancient Americans would write, for example 2 Ik O Pop, indicating that it was the 2nd day in the sequence of 13 numbers, that it followed the day, "Imix," in the series of 20 names, that it was the first day of the first month, "Pop."

We will next consider the number of days that must elapse before a day like "2 Ik O Pop," will be repeated with the same 4 symbols. After a tzolkin (260 days) has elapsed "2 Ik" is repeated and after 365 days "O Pop" reoccurs. Hence, a day bearing the designation, "2 Ik O Pop," will be repeated after a number of days which is the least common multiple of 365 and 260 or 18,980. This period is called the "Calendar Round" and, as the reader can readily verify, consists of 52 years or 73 tzolkins. This "Calendar Round" included all possible combinations of the 260 days with the 365 positions of the year. Morely describes this most distinctive feature of Mayan time reckoning as follows: "The fundamental principle of Maya chronology is that all periods from the lowest to the highest are always in continuous permutation, each returning unto itself and beginning anew after completion."

The "Calendar Round" might seem a logical unit to use in fixing the large time units of a date but the Mayans wanted a more convenient number for a base than 52. For the actual recording of dates the Mayans used a place value notation in which 20 was employed as a base except in the 3rd place. Here is the major part of their table of measure:

1 kin equals 1 day
20 kins equals 1 uinal equals 20 days
18 uinals equals 1 tun equals 360 days
20 tuns equals 1 katun equals 7200 days
20 katuns equals 1 baktun (or cycle) equals 144000 days.

A Mayan date was recorded as a number of 5 periods: baktuns

(or cycles), katuns, tuns, uinals and kins. This is followed by one of the 18,980 dates of the "Calendar Round." These 5 periods showed the number of days after a certain fixed date ("4 Ahau 8 Cumhu"), a reference point similar to the birth of Christ. A few examples of Mayan dates, in round numbers, are given by Spinden. These are "12-0-0-0-0 5 Ahau 13 Zotz," "7-0-0-0-0 10 Ahau 18 Zac," and "9-0-0-0-0 8 Ahau 13 Ceh." The first of these three dates signifies a day 12 baktuns, O katuns, O tuns, O uinals and O kins after the fixed date. This date is the 5th number in the "1–13" series, follows Cauac in the sequence of 20 day names, is the 14th day of the month of Zotz. The definiteness with which a Mayan date was fixed is summarized by Verrill. "If the numeral were appended the date was fixed within 260 days. If the month sign and its numeral were added it was fixed within 52 years. If the tun sign was given it was fixed within 936 years. If the katun sign appeared the date was fixed with 18,720 years, and if the baktun or cycle symbol were added the exact position of any day was fixed within a period of 374,400 years—a most astonishing mathematical achievement."

It is to be noted that many of the time periods of the Mayan calendar have no counterpart in nature but are essentially arbitrary, for example, the "1–13" series, the tzolkin and the 52 year period. This fact, and the consistent use of elapsed time in recording dates are outstanding characteristics of the Mayan calendar.

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Scientific Investigations of the Teaching of High School Mathematics Reported in 1933

By H. E. Benz
Ohio University, Athens, Ohio

I. Introduction

About two years ago the present writer prepared a summary of the investigations of the teaching of high school mathematics through 1931. This summary was brought through 1932 in an article in this journal last October. The present report continues this summary by reporting similar studies published during 1933.

II. The Mathematics Curriculum

In an effort to set up a defensible list of specific objectives for each of the three grades of the junior high school, Schorling (12) combined the list summarized in his "Tentative List of Objectives in the Teaching of High School Mathematics" with a list published in a report of the sub-committee on junior high school mathematics prepared for the North Central Association of Colleges and Secondary Schools. For each grade the objectives are listed under six headings: vocabulary, principles, skills, formulas, concepts, attitudes. In an extended discussion of high school mathematics Beatley (1) presented the results of an analysis of replies to a questionnaire sent to leading teachers of mathematics in high schools and colleges. He collected opinions relative to several crucial issues in the high school mathematics curriculum. Many replies are quoted in part, and certain proposals for the improvement of the curriculum are drawn from the interpretation of the replies.

III. The Present Status of Mathematics Teaching

In connection with the National Survey of Secondary Education made by the Office of Education of the United States Department of the Interior, Lide (8) made a study of instruction in mathematics as revealed by courses of study and by visits to certain high schools. This report represents a rather careful study of the status of mathematics teaching in the United States at the present time.

IV. The Organization of Subject Matter

Previous summaries similar to the one here reported contained a reference to studies which might appropriately be classified under the title above. No outstanding studies were discovered during 1933 which were concerned with the organization of subject matter.

V. Conditions Affecting or Accompanying Teaching

The study of mathematical ability continues to occupy the attention of students of teaching. Cooke and Pearson (5) studied the prognostic value of certain measures for predicting success in plane geometry. Subjects included 195 pupils in 10 different high schools. They concluded that the Orleans Geometry Prognosis Test was "not appreciably more accurate" for predicting success than the Terman Test or teachers' marks in algebra. They concluded also that the correlation between teachers' marks in algebra and performance in geometry was raised only very slightly by the introduction of the other two factors into the prediction. But in any case the correlations were too low to be useful. Important factors which condition success in geometry seemed to have escaped measurement. The problem of predicting algebraic ability was attacked by Dickter (6). He concluded that a composite of the Rogers Test of Mathematical Ability and teachers' marks for eighth grade mathematics "can be considered the most reliable and the most practical measure for predicting ability in algebra." The R obtained was .73. Achievement in algebra was measured by the Breslich Algebra Survey Test. A committee of the National Council of Teachers of Mathematics is working on the problem of the adaptation of mathematical subject matter to the needs of pupils. A preliminary report of this committee (11) describes the construction and evaluation of a unit on angles intended for seventh grade pupils of below-average ability. The unit was worked out with special attention to the psychology of dull pupils. The results indicate that dull pupils can master much worthwhile material in the field of mathematics when it is well organized. The matter of individual instruction in high school algebra was studied by Gadske (7). He used two groups equated on the basis of I. Q., arithmetical ability, and reading ability. The experimental group used unit assignments and individual instruction while the control group used group instruction. The author calls the method

used by the control group the "individualized unit method." There were 23 pupils in each group. At the end of the year the mean score on the Columbia Research Bureau Algebra Test, Form B, for the experimental group was 7.6 points higher than the mean score of the control group. This difference is almost four times its standard error. The author notes that the difference in performance is especially noticeable among the brighter pupils. Christofferson (4) made a significant contribution to the training of mathematics teachers in his study of professionalized subject matter for geometry. He set up a proposed program of mathematical and professional training for prospective teachers of high school geometry, which includes a body of material intended to give teachers a broad foundation of subject matter, especially along lines commonly overlooked in typical courses in college mathematics, and to give them a review of the material they will be expected to teach.

VI. The Learning of Mathematics

A study of demonstrative geometry in the ninth grade was made by Orleans (9). Three groups of ninth grade algebra pupils, selected on the basis of a prognosis test worked out for the purpose, were given, in addition to the regular algebra material, the Orleans unit on demonstrative geometry as given on pages 44 to 53 in the Fifth Yearbook of the National Council of Teachers of Mathematics. It is reported that their algebra did not suffer. The average grade for the work in geometry was above a B. The pupils represented a "high" group. The teachers assisting in the experiment believed that a "normal" algebra class could not spare the time from their algebra to work out the geometry unit. Orleans and Orleans (10) made a rather interesting study of the ability of pupils to learn without instruction. They gave the Orleans Algebra Prognosis Test and the Orleans Geometry Prognosis Test to pupils about to begin the study of these subjects before they were given any instruction whatever. Six weeks later the tests were given again. This procedure was followed out with two different classes, one in February, 1932, and one in September, 1932. It will be remembered that each section of these tests is preceded by a "learning unit" from which pupils are able to learn some of the important principles of the subject. The results indicate that pupils are able to learn significant amounts of algebra and geometry in this manner without

the teacher, and that by comparison the amount learned as a result of six weeks of instruction by the teacher is quite disappointing. The committee on geometry of the National Council of Teachers of Mathematics (2) presented its second report during 1933. In this report Professor Beatley described the work of the committee. The first step was to collect suggestions concerning the teaching of mathematics from periodicals, reports and books. These suggestions are summarized in the report. The committee's next step will be to "formulate a few distinct philosophies of the teaching of geometry" which will later be subject to experimental trial in cooperation with the schools.

VII. Problem Solving

No studies were found which dealt primarily with problem material or the teaching of problem solving in high school mathematics.

VIII. Instructional Materials

A study of a type of geometry material not ordinarily taught in high school courses was made by Bradley (3) in his study of the geometry of repeating design. This is a compilation of material in this field which will be new to most high school pupils and some high school teachers. The material is organized and interpolated into a conventional geometry sequence.

IX. Testing and Tests

In an effort to evaluate the influence of the College Entrance Board Examinations, Whitcraft (14) (15) made an extensive study of these tests. He analyzed the questions used, and then analyzed textbooks and courses of study to determine what effect the examinations had on the construction of instructional instruments. Teachers, superintendents, curriculum specialists, and other educators were questioned as to the influence of the tests. The general conclusion drawn is that the influence has been very great and, for the most part, undesirable. Schuck (13) studied the effect of the repetition of a mathematics examination. The Iowa Mathematics Training Examination was given to high school seniors in Minnesota in the spring of 1933. Eighty-three of these entered the Engineering School of the University of Minnesota in the following fall and repeated the test. The scores were noticeably higher

the mean of the raw scores rising from 44 to 52, the 75 percentile from 50 to 59, and the 25 percentile from 39 to 45.

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Joseph Louis Lagrange

Born at Turin, January 25, 1736
Died in Paris, April 10, 1813

In the case of Lagrange, we have a man whose interest seemed to lie in other fields until, apparently by accident, his attention was drawn to mathematics which he then studied almost to the exclusion of everything else. Lagrange was of French descent, but for a number of generations his people had been in Turin where representatives of his family held a certain political post in the Sardinian government. Loria notes that this position remained in the family until 1800 from which one concludes that it was abolished when Napoleon's campaigns changed the political organization of Italy.

It was hoped that Lagrange would become a lawyer and perhaps build up the family fortune which had been dissipated by speculation, but, instead, when he entered the university of Turin, his great interest was in physics.

At the age of seventeen, he came upon a memoir written by Edmond Halley. The title was "On the excellence of the modern algebra in certain optical problems." This was the means of turning his energies from physics to mathematics. He began to work independently in this subject with such success that he was appointed lecturer in mathematics in the artillery school at Turin in 1755. Ball says of this period:[1]

> The first fruit of Lagrange's labours here was his letter, written when he was still only nineteen, to Euler, in which he solved the isoperimetrical problem which for more than half a century had been a subject of discussion. . . . Euler recognized the generality of the method adopted, and its superiority to that used by himself; and with rare courtesy he withheld a paper which he had previously written, which covered some of the same ground, in order that the young Italian might have time to complete his work, and claim the undisputed invention of the new calculus (i.e., the calculus of variations). The name of this branch of analysis was suggested by Euler. The memoir at once placed Lagrange in the front rank of mathematicians then living.

In 1758, Lagrange founded a society which later became the Turin Academy in whose transactions he published memoirs that were the first of a long series. It was during his eleven years at

[1] *Short Account of the History of Mathematics*, 1915 ed., p. 402.

Turin that Lagrange made a trip to Paris where he met Clairaut, d'Alembert, and Condorcet all of whom were to be his associates at a later date.

In 1766, Lagrange succeeded Euler at Berlin. In urging him to come, Frederick the Great claimed that "the greatest king in Europe" should have the "greatest mathematician in Europe" at his court. The move was a successful one. To quote Ball again,[2]

Lagrange was a favorite of the king who used frequently to discourse with him on the advantages of perfect regularity of life. The lesson went home, and thenceforth Lagrange studied his mind and body as though they were machines, and found by experiment the exact amount of work which he was able to do without breaking down. Every night he set himself a definite task for the next day, and on completing any branch of a subject, he wrote a short analysis to see what points in the demonstrations or in the subject-matter were capable of improvement. He always thought out the subject of his papers before he began to compose them, and usually wrote them straight off without a single erasure or correction.

On the death of Frederick the Great, Lagrange determined to leave Berlin. He was urged to go to Spain, to Naples, and to Paris. He chose the last, taking up his residence in the Louvre in 1787.

His *Mechanique analytique* appeared in the following year. This work has been described as "a kind of scientific poem." In the preface, Lagrange called the reader's attention to the lack of diagrams in the book for he considered mechanics a branch of pure mathematics. For two years after the publication of this great work, Lagrange was too depressed to even glance at the printed copy. The cause of his melancholy seems to have been ill health, but a person wonders whether after all it might not have been closely allied to conditions in France. To come from the comparative security of the court of Frederick the Great to the hazards of that of Louis XVI just before the beginning of the French Revolution, must have been a nerve wracking thing.

Lagrange, however, was respected by each of the revolutionary governments in turn. He was appointed on the commission that developed the metric system and served on this through its work. When foreigners were banished from France, Lagrange was specifically exempted from the decree, but after the executions of Bailly and Lavoisier, he determined to leave the country. Of Lavoisier, he said that the guillotine had removed a head that it would take a century to reproduce. Lagrange was persuaded to stay, however,

[2] *Ibid.*, p. 404.

by an appointment to the newly organized Ecole normale in 1795. His career here was short. The work was elementary and the instructors were subject to constant surveillance. For example the instructors were pledged "neither to read nor to repeat from memory," and transcripts of the lectures as they were delivered were given inspected by the authorities so that all might be kept in the proper tenor.

In 1797, Lagrange became professor at the Ecole polytechnique where he seems to have been given more freedom, and certainly where the work was of a more advanced character.

Lagrange continued in favor when Napoleon came into power and he was given a title and suitable honors.

The reader is referred to Ball and to Cajori for summaries of Lagrange's contributions to mathematics for these lie beyond the elementary field. Their number and scope is tremendous. In one interval of twenty years, Lagrange is said to have written a memoir a month on an average.

It is possible that the reader may conclude that Lagrange was a mathematical vicar of Bray, or else, perhaps an astute politician. It is well, then, to conclude with the characterization which Cajori makes of him,[3]

> Lagrange was an extremely modest man, eager to avoid controversy, and even timid in conversation. He spoke in tones of doubt, and his first words generally were, 'Je ne sais pas.' He would never allow his portrait to be taken, and the only ones that were secured were sketched without his knowledge by persons attending the meetings of the Institute.

[3] *A History of Mathematics*, 1926 ed., p. 259.

VERA SANFORD

Members of the National Council

DO NOT miss the Pittsburgh meeting of the Council in December. The program of the meeting will be found on the first page of this issue.

◆ NEW BOOKS ◆

Relational and Functional Thinking in Mathematics. By Herbert Russell Hamley, Ph.D., Bureau of Publications, Teachers College, Columbia University, 1934, 8+215. Price $1.75 postpaid.

This is the ninth Yearbook of the National Council of Teachers of Mathematics, and if the organization had published nothing else it would by this single volume have amply justified its existence. For here is a work of such scholarship and sound judgment as to make it rank among the best that America or Europe has produced in our generation in the field of education as applied to elementary mathematics. This is high praise, but it is justified. Dr. Hamley has not merely set forth the opinions of American writers, but he has read and digested the leading authorities in this country and also in Germany, France, Austria, Hungary, Italy, Spain, Russia, and England. Rurthermore, and this is of great importance, he has not been content with quoting the words of as many writers as possible. On the contrary, he has sought only the essence of the contributions of the leaders in the modern reform movement —of men who can think for themselves, and act. It is frequently, and indeed generally, the case that those who are sympathetic with the reform movement judge their contributions by the mass of undigested quotations which they try to absorb; Dr. Hamley has the rare power of selecting his intellectual food and then of digesting it.

It will be an unusual experience for our American teachers to read the chapter on The History of the Function Concept. We usually think of this phase of modern teaching as due to Felix Klein, and indeed it is to him more than to any other leader that we owe its introduction in the secondary school. Dr. Hamley tells us, however, that the idea goes back to one of the distinguished von Humboldt brothers early in the nineteenth century, and he traces briefly its development in Germany, showing the influence of that country on the whole reform movement of which this theory has become an essential part—indeed, "the heart and soul of mathematical thinking." He also directs attention to the great influence which Klein and other German scholars exerted through the International Commission on the Teaching of Mathematics. Having been closely associated with Klein for six years, while this work of the Commission was being carried on, this reviewer can testify to his clear insight into the problem and his vigorous activity in making the function concept known as a school element in most of the leading countries of the world. It is not too much to say that no mathematician of the first rank has in modern times exerted more influence than he upon the teaching of mathematics in the secondary schools. If any other name deserves mention as his equal it is that of Jean-Gaston Darboux who died in 1917. In the efforts of this scholar to introduce the idea of the function, and thereby to place the calculus in the secondary school, he was ably supported by Jules Tannery, the active head of the École normale supérieure. We are only just beginning, in America, to appreciate the sound arguments of

these three men. If the calculus were rationally taught, it could easily replace many of the relatively unimportant parts of the mathematics in our high schools today.

It would naturally be expected that Dr. Hamley would trace with exceptional insight the reforms in the teaching of mathematics in England. Closely associated with me like C. S. Jackson, Sir Percy Nunn, C. Godfrey, E. H. Neville, and G. St. L. Carson, he could hardly fail to enter into the movement with an unusual equipment.

Chapter V, "The Function Concept in the Secondary School," is of great value to any teacher who is interested in this subject—which should include every teacher in this country. The chapter discusses some of the most important of the recent textbooks in Germany, France, England, and other countries, including our own. Chapter VI considers "The Function Concept in Practice," and the book closes with "A Course Based on the Function Concept" and "Tests of Mathematical Relations." To the teacher the first of these will be found of special importance and assistance. It may be felt that Dr. Hamley here tends to be extreme in his views, but this is natural in treating of a single phase of the teaching of any subject, and in any case he has accomplished what few others have even attempted—he has given us something beside loose talking; he has set forth a definite procedure, one which may readily be merged with the traditional course by any skilled teacher.

As in all reforms there lurks some danger in the carrying out of the project. There is always a tendency to make an easy task hard by using language that discourses the hearer. To talk about "functional thinking," "algebra as a mode of thought, "geometry as the science of necessary conclusions," and the like, when the pupil's vocabulary is not ready for such expressions, is simply an attempt to display our erudition—an erudition which is often a purely "imaginary quantity." Indeed, if a teacher should fall into the error of attempting to define such terms as "variable," "function," "functionality," "functional relationship," "class," "order," "continuity," "limit," and "functional thinking," or even to use the terms until the pupils are well along in mathematics, the presentation of the function concept would be doomed to failure. It would be like trying too early (if ever) to define "arithmetic," "algebra," "geometry," "angle," and many other words which gradually acquire a meaning when the mind is prepared to understand it, even though imperfectly. Nevertheless, for the teacher who undertakes the reform which a phrase like "functional thinking" suggests, the reading of the first three chapters of this work will prove both informational and stimulating.

The book is worth owning, worth reading, worth pondering, and worth assimilating.

David Eugene Smith

A History of Mathematics in America before 1900. Carus Mathematical Monograph, Number Five. David Eugene Smith and Jekuthiel Ginsburg. Open Court Publishing Company, Chicago. 209+x pp. Price

On first eight, the title of this volume makes one ask whether mathematics had any significant history in America before 1900. In the Introduction, Professor Smith gives his defense of the subject. He says, "Even a brief examination of the question, however, shows not only that the work done before the year 1875 is worthy of attention, but that the succeeding quarter of a century saw laid the

foundations upon which the scholars of today have so successfully built."

The authors have confined themselves to the territory now included in the United States and the Dominion of Canada, but to all intents and purposes the work is virtually limited to the former. The reader is referred to the discussion of the work on mathematics in the Spanish speaking countries by the late Professor Cajori, to the work on early American arithmetics by Professor L. C. Karpinski, and to the work on algebra in American schools in the eighteenth century by Professor Lao G. Simons. The book is illustrated by portraits of John Winthrop, Robert Adrain, Benjamin Peirce, J. J. Sylvester, E. H. Moore, Maxime Bôcher, and J. Willard Gibbs.

The sixteenth and seventeenth centuries are treated briefly in a single chapter. The eighteenth century is given greater space. The nineteenth century is considered first in a general survey and then in a section devoted to the period from 1875 to 1900 which comprises half of the volume.

The style of the work and the scope of the treatment can be judged from the following summary which appears on pages 197 to 199.

"From 1500 to 1600 the aims and achievements [in mathematics] were hardly commensurate with those of a mediocre elementary school of our time. From 1600 to 1700, they were not even equal to those of our high schools of low grade, but the purpose was more definite than in the preceding century. The objective was now to give to those seeking it enough work in astronomy to predict an eclipse and to find the latitude of a ship at sea, and enough mensuration to undertake the ordinary survey of land.

"From 1700 to 1800 the general nature of the work in the colleges was that found in the two great universities of England, but it was far from being of the same quality. The courses then began to include algebra, Euclid, trigonometry, calculus, conic sections (generally by the Greek method), astronomy, and "natural philosophy" (physics). The prime objective was still astronomy. . . .

"From 1800 to 1875 America began to show a desire to make some advance in both pure and applied mathematics, independently of European leadership. The union of mathematics, astronomy, and natural philosophy was still strong, and the pursuit of mathematics for its own sake was still somewhat exceptional.

"From 1875 to 1900, however, a change took place that may well be described as little less than revolutionary. Mathematics tended to become a subject *per se;* it became "pure" mathematics instead of a minor topic taught with astronomy and physics as its prime objective. American scholars returning from Europe brought with them a taste for abstract mathematics rather than its applications. There were many exceptions. . . . Nevertheless, the tendency was strongly toward pure analysis and geometry."

VERA SANFORD

Differential and Integral Calculus. By Clyde E. Love, The Macmillan Company, 1934. 15+383. Price $2.75.

This is a new edition of a popular calculus text which includes an introduction to the more elementary phases of differential equations in the last few chapters.

The new edition is similar to the old but the explanatory materials and worked out examples have been increased in number to facilitate ease in reading and a more complete understanding of the subject.

A brief treatment of the approximate

solution of equations has been added, the section being, in effect, a short exposition of Newton's method of approximate solution.

The treatment of the integral calculus has been moved forward and the sections on indeterminate forms and curve tracing correspondingly postponed. This treatment seems to improve the continuity of the book.

As before, the number of practical applications in problem form, of the calculus to the natural sciences remains one of the outstanding features of this text.

CHARLES RUSSELL ATHERTON

Exercises and Tests in Intermediate Algebra by D. E. Smith, W. D. Reeve, and E. L. Morss, Ginn and Company, 1934, is "designed to furnish material which can be used for drill and test purposes in any modern high-school or college course in intermediate algebra. Care has been taken to provide an adequate review of elementary algebra which will not only enable the teacher to measure the student's ability to apply skills already acquired and to diagnose weaknesses, but will also enable the student better to appreciate algebra as a *method of thinking*."

This workbook shows evidence of being thoroughly modern and scientifically constructed. Processes and manipulations which are unquestionably obsolete are not included. This leaves more room to provide abundant drill and emphasis for those processes and techniques which are needed in later work in science and mathematics. Then too, the tests and exercises serve admirably for diagnostic purposes. The individual weaknesses of pupils in a class are readily disclosed and identified, thus providing for effective remedial teaching and drill.

Unlike many workbooks designed largely as test and drill exercises, there is in this book ample recognition of some of the more elusive objectives of algebra such as building formulas and developing the idea of dependence. Furthermore, teachers concerned with preparing students for College Entrance Examinations will find carefully constructed materials to provide experience with the type of exercise usually called for on these examinations. In general, this book is an inexpensive, comprehensive, carefully constructed series of well organized review exercises and new materials, and as such should prove very useful for teachers of Intermediate Algebra.

H. C. CHRISTOFFERSON

Instruction in Mathematics Bulletin, 1932, No. 17, Edwin S. Lide, National Survey of Secondary Education Monograph No. 23 Washington, D.C., 1933.

Mr. Lide presents the following conclusions:

1. *With regard to materials of instruction and the adaptation of such materials to the needs of pupils, practices in junior high schools appear much more outstanding than those in senior high schools.*

One of the major aims of mathematics in the early period of the secondary school is to enable the pupil to find out where his interests lie and what his abilities are, and to satisfy his everyday mathematics needs. It is gratifying to find evidence that the secondary schools are beginning to accomplish this. They are organizing instructional materials of the early periods in relation to the pupils' interest and ability. This seems to be especially true in the courses which comprise the required work. Mr. Lide reports a tendency (21) to require two years of mathematics, i.e., the mathematics of grades seven and eight. This is considerably less than what was

required twenty to twenty-five years ago and falls below the more recent recommendation of the National Committee which recommended a requirement of three years junior high school mathematics.

Simultaneously with the reduction, the character of the work offered in grades seven and eight has changed from arithmetic to unified methematics. He finds that only one-third of the schools examined still designate the work as "arithmetic," the others having changed to courses in "correlated' or "general" mathematics. Even in grade nine the number of schools designating the course as general mathematics is the same as the number listing it as algebra.

Moreover, when the seventh-grade courses labeled arithmetic (25) were analyzed, it was found that they were not actually courses in arithmetic. Three of the 12 major topics were geometric, one with graphs, three with applications to banking, business, and the home, and four with the fundamental processes with integers and fractions. Eighth-grade courses labeled arithmetic contained seven topics on algebra and geometry and five on arithmetic and its applications. Thus, the practices disclosed in 57 courses of study outlines of schools selected because they were reported to be doing outstanding work show a marked tendency to unify in the early stages the various mathematical subjects. This conforms to the recent tendencies in education toward integration of school subjects.

2. *It is evident that for the selected groups of schools being considered practical and utilitarian aims of mathematics in grades seven, eight and nine are considered of most importance* (21). This conclusion was derived from a tabulation of objectives (19). Objectives were listed as practical, disciplinary and cultural. In the practical were included accuracy and facility in the fundamental processes, knowledge and power to apply mathematical concepts, specific knowledge useful in life, and exploration and guidance. Many of the objectives listed possess not only practical but also disciplinary and cultural values. This applies to such objectives as understanding of fundamental laws, accuracy in problem solving, power to apply and development of a number sense. Many teachers rate such work high, not because it has practical use but because of the training that may be derived from it. Mr. Lide classifies 58% of the objectives as practical, 20 as disciplinary and 19 as cultural. A different classification will alter these percentages considerably and also the conclusions. Most writers ranks the disciplinary objectives above the practical.

3. *Teachers seem much dependent on a single adopted book.* Those who visit teachers frequently will probably agree with Mr. Lide. They will hardly agree with the solution suggested by the Boston teacher who would discard the textbook. A better suggestion would be to use more than one book. In other subjects this has become a practice years ago. Pupils in mathematics must be taught correct habits of reading and study or they will become dependent on the teacher and their possibilities for self-development will be seriously endangered. The pupil who knows how to study his textbook has a tremendous advantage over the one who has been deprived of such training. Even the teacher who merely teaches a book will probably do better with it than without it.

In a modern course in mathematics one should go beyond the textbook. It should be supplemented by a mathematical library to which pupils should

go for additional inspiration and information. Experience has shown that a mathematics section in a library will attract as many and more readers than an English or history section.

4. *The greatest problem in junior high school seems that of adapting materials in grade nine to college and non-college pupils.* With the tendency of requiring only two years of mathematics and of providing for the mathematical needs of the non-college pupils during these two years, this problem should not trouble the teachers very much longer. Most non-college pupils should take up non-college mathematics such as are offered in industrial, commercial and vocational courses. The future college pupil must recognize preparation for future mathematics as a major objective. If he cannot absorb much work he is unlikely to be good college material. However, it is a long way from the ninth grade to the college and it is difficult to see why the best instruction the teacher is able to perform is not at the same time the best preparation for college work. Whenever the problem assumes serious dimensions the best solution is probably to break away from traditional algebra and to replace it by some mathematical offering which is interesting and profitable to all pupils, the college preparatory group and those who do not care for the industrial and commercial mathematics but wish to continue the study as long as they remain in school.

5. *Probably the greatest need is a city-wide program for measuring the results of instruction.* Mathematics was one of the first subjects in which the testing movement was accepted and encouraged. First the standard arithmetic test appeared. Then followed tests in algebra, geometry and the higher subjects. The performance type test received attention first. Much good work was accomplished which served as patterns for developing tests in other subjects. When demands for different types of tests arose mathematics was again assuming leadership. Mathematical ability tests, prognostic tests, survey tests and other types were developed and much creditable work was done.

Now, with these aids available it would be unfortunate that teachers fail to make use of them. It is possible that the method employed by Mr. Lide failed to disclose the actual facts about the measurement of results. For, publishers sell a tremendous amount of test material which is being used by mathematics teachers.

In addition to the standardized tests other types have recently come into wide use. The inventory test, the practice test, the improved class examinations on the various units of the course accomplish results which cannot be attained by the standardized tests. They do not replace but supplement them. They are closely related to the content of the course, have diagnostic value, and can be followed up with remedial instruction. Such tests are inexpensive because they are usually made by the teachers themselves. Mr. Lide's conclusion for departmental and city-wide testing programs (40) deserves serious consideration on the part of the teachers of mathematics.

6. *Method of analysis of textbook.* Mr. Lide refers to several studies aiming to determine tendencies in the content of the mathematics curriculum. One such study analyzed 257 textbooks published during a ten-year period. It should be pointed out that length of time and number of textbooks are not alone sufficient to obtain reliable findings. The method of analysis is the more important. Many investigators start out with a list of traditional topics, proceed to analyze books with reference to them, compute percentages and draw conclu-

sions. The findings will discover only tendencies as to the traditional materials. New tendencies are likely to be overlooked. What is needed is an analysis of books with regard to new materials or different materials. If the original list does not include functional thinking the results can not disclose a tendency to emphasize functional thinking. It is less important to know that the modern tendency is to include as much work on fractions as was the practice ten years ago than to determine whether the character of the fractions has undergone changes. Thus, the nature of the fractions in the college entrance examination has been greatly simplified in the last twenty-five years. What we need to know is not how much space is given to fractions in textbooks but whether there is a corresponding tendency toward simplification.

7. *Acquisition of objectives.* Mr. Lide reports attempts of various teachers to attain certain objectives. Thus, in one school the teachers try to attain the "accuracy" objective. To develop accuracy in geometry pupils are required to make accurate constructions. Incidentally it is mentioned that they make the constructions on the blackboard with string and chalk. It is a mistake to assume that accuracy can be attained by talking about it. The least that should be done is to use the best instruments in making constructions, in which case compasses should be used. It should be pointed out to the pupils that absolute accuracy is impossible, that the making of constructions requires the use of compasses, and that when less accurate drawings are sufficient, string and chalk may serve the purpose. However, this distinction is not always made in teaching. It is essential if construction work is to contribute to the accuracy objective.

E. R. BRESLICH

Mathematics Essentials For Elementary Statistics. By Helen M. Walker. Henry Holt and Co., 1934. XIII+246 pp.

This book was written for the adult layman who has an interest in studying statistical method, but who finds that his background of elementary mathematics necessary for such study is hazy and uncertain.

The book is the result of several years of concrete experimental investigation on the part of the author who has been interested in presenting statistical method in such a way that intelligent students of mature years may not only understand but enjoy the study of statistics as a branch of applied mathematics.

The scope of the book includes the mathematics necessary for the understanding of any statistics text likely to be used in a course for which the calculus is not a prerequisite. It presupposes very little knowledge of mathematics, even giving a review of certain arithmetic topics ordinarily taught below the eighth grade. On the other hand, it includes material, algebraic in nature, but unfamiliar to the average teacher of secondary mathematics.

The content of the book is organized on two levels. The first part furnishes a basis for the more elementary courses in statistical method and the second part presents more difficult material designed to give an adequate background for the study of more advanced work.

Elements of Analytic Geometry. By C. E. Love. The Macmillan Co., 1932. XI +149 pp., $1.60.

This text is an abridgement, rather than a revision, of the authors *Analytic Geometry* and is intended for uses in courses where the author's earlier text cannot be satisfactorily covered for lack

of time. Reduction of content is made largely in the material dealing with the treatment of the properties of conics and the discussion of surfaces in space. Certain topics have been omitted entirely as have been the more difficult exercises, but the development or explanation of necessary topics and required drill material have not been omitted.

The Elements of Euclid. By Isaac Todhunter with an introduction by Sir Thomas Heath. E. P. Dutton & Company, 1933. XVIII+298 pp., $0.70.

It is a little unusual for one to think of *Euclid's Elements* being read as a literary textbook, but this is the purpose in the minds of the publishers and Sir Thomas Heath. This Todhunter edition of Euclid's is based upon Simpson's and is presented as the most suitable edition to reprint in *Everyman's Library*.

This edition ought to appeal to all persons of culture except the few who are not inclined toward the study of mathematics.

Progressive First Algebra. By W. W. Hart. D.C. Heath and Company, 1934. VI+408 pp. $1.28.

This additional text of a wellknown author provides a unit organization of material by pages and is so organized as to bring out the inductive type of instruction in such a way as to furnish training in the scientific method of thinking.

The core material is *formulas, equations, problems, functional relations, graphs, numerical trigonometry, and simplified algebraic technique*. Provision for children of different abilities is made by marking examples and topics by Y or X.

Diagnostic texts, chapter mastery tests, short tests, reviews, and new type comprehensive tests are included.

Analytic Geometry. By F. S. Nowlan. McGraw-Hill, 1934. XII+352 pp. $2.25.

This text is designed primarily for use in first- and second-year university classes. It is the outgrowth of many year's experience in teaching analytic geometry by the author in American and Canadian Universities to art and engineering students. The book is intended to be analytic in character and strictly rigorous.

Extensive use is made of orthogonal projection in order to obtain in a simple manner generality of demonstration. As a result the idea of a locus and of the equation of a locus are stressed as fundamental elements.

Teachers of analytic geometry will be interested to see this new addition to the present stock of texts in analytic geometry.

Progressive Second Algebra. By W. W. Hart. D. C. Heath and Company, 1934. IV+298 pp. $1.32.

This is the second course of the author's algebra series and it is designed for the third semester of algebra as it is given in most high schools.

The first seven chapters constitute a complete review of the first course in algebra which may be good or bad depending upon the way the teacher uses it. If it used as pure repetition the result is likely to be bad. If the material is presented as a new view of old material it can be made helpful.

The remaining chapters provide a course that will equip students who wish to qualify for the examinations of the College Entrance Examination Board and with the recent recommendations of the National Committee on Mathematical Requirements.

Provision for individual differences is made and the unit organization of material is made.

Essentials of Plane Trigonometry and Analytic Geometry. By A. H. Sprague. Prentice-Hall, Inc., 1934. X+228 pp. $1.80.

This book has been written to meet the needs of those students who may need to prepare for a study of the Calculus. Although the material is combined in a simple volume the distinction between the subjects of trigonometry and analytic geometry is brought out in the arrangement of the material. Thus, the second part of the book—supplemented by the earlier sections on coordinate systems, found in the first part can be used as a separate course in analytic geometry where a previous knowledge of trigonometry is assumed.

This book will be welcomed by teachers who wish material so arranged.

Essentials of Plane Trigonometry. By A. H. Sprague. Prentice-Hall, Inc. 1934. VIII+124 pp. $0.80.

This work is written with the idea of presenting in as simple a manner as possible, without the loss of rigour, the essential elements of a reasonable short course in Plane Trigonometry.

The book begins with a chapter on logarithms and exponents and defines functions of an acute angle first so as to familiarize the student with the subject through applications before functions of a general angle are introduced.

The fundamental formulas and identities are given in a single chapter and a large amount of problem material on the solution of identities is included in the book.

This book should be of interest to both secondary and college teachers of trigonometry.

NEWS NOTES

WHAT CONSTITUTES a general education and how to achieve it occupied the attention of some 250 educators of the country attending the University of Chicago's ninth annual Institute for Administrative Officers of Higher Institutions, which was held this summer in Judson Court on the Midway. The subject of the conference was: "A New Definition of General Education."

"There is a growing tendency toward confusing the art of living and the art of getting a living," Henry W. Prescott, professor of classical philology and chairman of the department of Latin at the University of Chicago, told the Institute in the morning discussion of the "Nature, Scope, and Essential Elements in General Education."

He cited two recent government reports on actual practice in the school system as evidence that educational practice is seriously affected by the feeling that schools must be adapted to the communities which they serve and to the needs of individual students. "The two reports," he said, "show that in a Los Angeles suburb composed almost exclusively of children of aliens, and in Evanston, Illinois, a community of different type, the trend is largely toward vocational efficiency and preparation for living on the lower levels.

"As in Los Angeles, so in Evanston, the girl student in home economics probably will wear prettier hats and cook better the family food, or become a proficient milliner or baker; the boy in automobile mechanics will more expertly drive and repair his machine or profitably pursue the trade of automobile mechanic," he said. "In spite of the wide difference between the Los Angeles suburb and Evanston, the arts of living and of getting a living are similarly combined or confused in both places.

"Is the art of living in this country of ours so different from the art of living in England, France, Germany, Belgium, Italy that the needs of communities, however heterogeneous, and of our children, however variously constituted, must be passively served by a chaotic mess of subject matter in which money-making, citizenship, bodily health, and mere acquaintance with environing conditions emerge as the only essentials, instead of consciously disciplined thought and the direct application of our thinking to some one or more of the various fields of worthy human activity?

"My program rests primarily on the thesis that the art of living happily and effectively is in large measure the same for all of us so far as education, properly so-called, may contribute to it.

"In my opinion the objective of general education should be, first, the development of clear thinking leading to intelligent action; second, the development of clear, convincing, and persuasive expression as the medium of expressing thought; third, the development of an imagination sensitive to the effects of literature, music, and the plastic arts; fourth, the knowledge and understanding of the past and the environment of the present in those respects that vitally affect intelligent activity in our present-day world. These four aims and in this order should determine the prescribed core of the curriculum through the entire years."

In contrast with this definition, the present trends mark a decided movement toward shallowness and superficiality, Professor Prescott contended. He objected particularly to the emphasis given the "so-called social studies, which are pale reflections of the social sciences, so-called, that lie above them in the university."

"In my view," Professor Prescott said, "The prescribed curriculum should include subjects in which the methods of valid reasoning are exemplified, fundamental laws established, and rigorous thinking constantly exercised. First in importance is mathematics; second, physics; third, various other sciences such as chemistry, biology, astronomy, geography. In such subjects, properly taught, the student is more likely than in any others to acquire habits of correct thinking.

"One of the curious paradoxes in contemporary education is that the technique of teaching has vastly improved in the direction of cultivating in the student independent thought, but almost all the subjects through which the practice of rigorous thinking is most easily inculcated have given way before purely descriptive material."

Professor Prescott advocated two units of social studies for those who leave school at the end of the twelfth grade, but only one for those who continue into the junior college, adding at the latter level a sequence in the political science, economics, and sociology.

Grammar, because it emphasizes the close interrelation of valid thought and clear expression; foreign languages, as a means of throwing into relief the structural elements that familiarity with the English language tends to blur, and also to prove a secondary key to the understanding of foreign cultures, the fine arts, were other items in Professor Prescott's recommended curriculum for the earlier stages of general education.

For the junior college he advocated courses in economics, politics, sociology, psychology, ethics and philosophy. The culmininating point of general education, he said, should be a course of study in logic or reflective thinking combined with and centered on material illustrating the development of significant ideas in the history of human thought.

Discussing the subject from the viewpoint of the social scientist, and from the aims of the social sciences general course in the University of Chicago's New Plan, Louis Wirth, associate professor of sociology, said that general education must not merely furnish the basis for civilized life but also lay the foundations for, and stimulate an interest in, further education.

"The purpose of general education is to give to the students who will, after they are graduated, go on living if not studying, a sense of the whole of modern thought which shall be sufficiently ordered and impressive that it will succeed in penetrating into whatever realm of life or thought or science with which they may become preoccupied," Professor Wirth told the Institute.

"One major problem in administering such a program is that of avoiding superficiality and dilletantism. In the social sciences general course a point of departure has been therefore, to avoid on the one hand making it a course in current events or current dogmas, and on the other hand to avoid repeating the dogmas of old as if they contained the answers to perennial and current problems of social life.

"Our general course in the social sciences has selected one major theme for intensive exploration, namely, the social, economic, and cultural consequences incident to the industrial revolution, which apparently transformed human life more profoundly than any other similar event in modern history.

"Because students come to the social sciences with the belief that they already have the right answers to all of the important questions, one task is to unsettle a great many aspects of their minds that were settled before," Dr. Wirth said.

"This exposes us frequently to the charge of indoctrination. The only answer we can make to this charge is that we are constantly making efforts to discover our own biases, and that nothing is more dangerous to assume in the social world than that we have arrived at absolute and final truths in which our own interests have played no role."

Mr. C. N. Mills of Illinois State Normal University recently sent The Mathematics Teacher the following note:

"On page 110 of the February (1934) ssue appears the *Men-Monkey-Coconut Problem*. The following information may be of interest to the readers of *The Mathematics Teacher:*

"This is the Ben Ames problem that appeared several years ago in a story in *The Saturday Evening Post*. The problem was proposed to the readers of *School Science and Mathematics*. It ran its course to a general solution. See April 1929, page 430; October 1929, page 771; December 1929, page 990 and May 1930, page 584."

At the recent meeting of the American Mathematical Society in Chicago, the dinner program was devoted entirely to honoring emeritus professors, Thomas F. Holgate of Northwestern University and Herbert E. Slaught of the University of Chicago. Each had been secretary of the Chicago Section of the Society for a ten-year period and each had served his University continuously for more than forty years. Professor Slaught had been made honorary president for life of the Mathematical Association of America, at its previous Cambridge meeting, in view of his leading part in founding and promoting that association and in reorganizing the American Mathematical Monthly in 1913, which was to become its official journal in 1916. Professor Slaught is also an honorary life member of the *Central Associations of Science and Mathematics Teachers.*

During the past year, Section 19 (Mathematics) of the New York Society for the Experimental Study of Education, held six dinner meetings with an average attendance of 47 at the dinners and 83 at the discussions later in the evening. These meetings were held at the Men's Faculty Club of Columbia University. On February 17, 1934, the members of Section 19 attended the open luncheon and meeting of The Mathematics Chairmen's Association at Hotel Astor, when Dr. Silberstein, Professor Schlauch, Mr. Lazar, and Dr. Astor were the guest speakers and Mr. Grady, Dr. Tildsley, Mr. Wright, Mr. Ernst, and Mr. Wilson led the general discussion.

The officers of Section 19 for 1934–1935 are:

Chairman: Professor W. D. Reeve, Teachers' College, Columbia Univ.

Assistant Chairman: Professor W. S. Schlauch, New York Univ.

Secretary: Alma Ekholm, Girls' Commercial High School, Brooklyn, N. Y.

The following outline shows the program of the dinner meetings for the past year:

October 14—Professor Reeve, Professor Schlauch, Mr. Swenson, Miss Meta Wood, "Mathematics and the Integrated Program."

November 18—Professor Mallory, Miss A. Loughren, Dr. Mac Mackin, Mr. Collins, Professor Hart, Mr. Strader, Mr. Kertes, Mr. Amidon, "Ninth-

Year Mathematics for Slow-moving Pupils."

December 16—Dr. John R. Clark, "Teaching Mathematics as a Method of Thinking."

January 20—Mr. John Swenson, "New Mathematics Versus the Old."

March 24—Mr. Joseph Schillinger, "The Mathematical Basis of Music and the Arts."

April 28—Mr. Claude Bragdon, "Art and Mathematics."

THE 51ST REGULAR MEETING of the Association of Mathematics Teachers of New Jersey was held at the State University at New Brunswick on May 5, 1934.

The following program was given:

Address—"Mathematics as a Way of Thinking or a Practical Tool," Dr. John R. Clark, Principal of the Lincoln School of Teachers College, New York.

Address—"Some Thoughts About Education for its Practical and Thinking Values," Dr. Ira T. Chapman, Superintendent of Public Schools, Elizabeth.

Address—"Maintaining in the Senior High Schools Skills in the Fundamentals of Arithmetic," Mr. Virgil S. Mallory, Associate Professor of Mathematics, Montclair State Teachers College.

Report—"A Course of Study in Ninth-Year Mathematics," Mr. Roscoe Conkling, Chairman of the New Jersey Mathematics Syllabus Committee.

THE SPRING MEETING of the Connecticut Valley Section of the Association of Teachers of Mathematics in New England was held at the Connecticut State College at Storrs, Conn. on May 26, 1934.

PROGRAM

Morning Session

10:30 "The Scope of an Elementary Slide Rule," Dr. Wm. Fitch Cheney, Jr., Connecticut State College.

"The Meaning of Mathematics," Mr. J. B. Hebberd, Suffield School.

"The Elements of Non-Euclidean Geometry." Dr. Rose Whelan Sedgewick, Connecticut State College.

12:30 Luncheon.

Afternoon Session

2:00 Business Meeting.

"Skew-Squares," Dr. Robin Robinson, Dartmouth College.

"Some Analogies Between the Tetrahedron and the Triangle," Dr. Joseph Rosenbaum, Milford, Connecticut.

Officers of the Connecticut Valley Section

Professor H. M. Dadourian... *President*
Trinity College, Hartford

Carroll G. Ross......... *Vice-President*
Mount Hermon School

Arthur D. Platt............. *Secretary*
Mount Hermon School

Miss Mary Noyes........... *Treasurer*
High School, Bristol

Walter Blaisdell.............. *Director*
Nathan Hale Junior High School

Miss Dorothy S. Wheeler..... *Director*
Bulkeley High School, Hartford

Past Presidents

Harry B. Marsh	'20
Percy F. Smith	'21
M. M. S. Moriarty	'22
Eleanor C. Doak	'23
Joe G. Estill	'24
Joshua I. Tracey	'25
Lyon L. Norton	'26
John W. Young	'27
Rolland R. Smith	'28
Harriet R. Cobb	'29
Melvin J. Cook	'30
Bancroft H. Brown	'31
Dorothy S. Wheeler	'32

Official Notice

As secretary of the National Council of Teachers of Mathematics, I officially announce the annual election of certain officers of the National Council, said election to take place at Atlantic City, New Jersey, on Friday, February 22, 1935. Article III Section 7, of the by-laws states: "At least two months before the date of the annual meeting, all members shall be given the opportunity through announcement in the official journal to suggest by mail for the guidance of the directors a candidate for each elective office for the ensuing year. At least one month before the annual meeting the secretary of the board of directors shall send to each member an official ballot giving the names of two candidates for each office to be filled. These candidates shall be selected by a nominating committee of the board of which the secretary shall be chairman. The election shall be by mail or in person and shall close on the date of the annual meeting."

At the Detroit meeting, 1931, of the National Council, the nominating committee consisting of the two most recent ex-presidents and the secretary as chairman (for this year: John P. Everett, William Betz, and Edwin W. Schreiber), was instructed to prepare a primary ballot suggesting five eligible candidates for each elective office. The officers to be elected at the Atlantic City meeting are: second vice president 1935 to 1937 and three directors 1935 to 1938.

The periods of service of the officers of the National Council, from its organization in 1920 to the present time, are printed below.

EDWIN W. SCHREIBER, *Secretary*

The National Council of Teachers of Mathematics

ORGANIZED 1920—INCORPORATED 1928

Periods of Service of the Officers of the National Council

PRESIDENTS

C. M. Austin, Oak Park, Ill., 1920

J. H. Minnick, Philadelphia, Pa., 1921–1923

Raleigh Schorling, Ann Arbor, Mich., 1924–1925

Marie Gugle, Columbus, Ohio, 1926–1927

Harry C. Barber, Exeter N. H., 1928–1929

John P. Everett, Kalamazoo, Mich., 1930–1931

William Betz, Rochester, N. Y., 1932–1933

J. O. Hassler, Norman, Okla. 1934–1935

VICE-PRESIDENTS

H. O. Rugg, New York City, 1920

E. H. Taylor, Charleston, Ill., 1921

Eula Weeks, St. Louis, Mo., 1922

Mabel Sykes, Chicago, Ill., 1923

Florence Bixby, Milwaukee, Wis., 1924

Winnie Daley, New Orleans, La., 1925

W. W. Hart, Madison, Wis., 1926

C. M. Austin, Oak Park, Ill., 1927–1928

Mary S. Sabin, Denver, Colo., 1928–1929

Hallie S. Poole, Buffalo, N.Y., 1929–1930

W. S. Schlauch, New York City, 1930–1931

Martha Hildebrandt, Maywood, Ill., 1931–1932

Mary A. Potter, Racine, Wis., 1932–1933

Ralph Beatley, Cambridge, Mass., 1933–1934

Allan R. Congdon, Lincoln, Nebr., 1934–1935

SECRETARY-TREASURERS

J. A. Foberg, Chicago, Ill., 1920–1922, 1923–1926, 1927, 1928 (Appointed by Board of Directors)

Edwin W. Schreiber, Ann Arbor, Mich., and Macomb, Ill., 1929– (Appointed by the Board of Directors)

COMMITTEE ON OFFICIAL JOURNAL

John R. Clark, Editor, 1921–1928

W. D. Reeve, Editor, 1928–

Vera Sanford, 1929–

H. E. Slaught, 1928–

DIRECTORS

Marie Gugle, Columbus, Ohio, 1920–1922, 1928–1930, 1931–1933.

Jonathan Rorer, Philadelphia, Pa., 1920–1922

Harry Wheeler, Worcester, Mass., 1920–1921

W. A. Austin, Fresno, Calif., 1920–1921

W. D. Reeve, Minneapolis, Minn., 1920, 1926–1927

W. D. Beck, Iowa City, Iowa, 1920

Orpha Worden, Detroit, Mich., 1921–1923, 1924–1927

C. M. Austin, Oak Park, Ill., 1921–1923, 1924–1927, 1930–1932

Gertrude Allen, Oakland, Calif., 1922–1924

W. W. Rankin, Durham, N.C., 1922–1924

Eula Weeks, St. Louis, Mo., 1923–1925

W. C. Eells, Walla Walla, Wash., 1923–1925

*Harry English, Washington, D.C., 1925–1927, 1928–1930

Harry C. Barber, Boston, Mass., 1925–1927, 1930–1932, 1933–1935

Frank C. Touton, Los Angeles, Calif., 1926–1928

Vera Sanford, New York City, 1927–1928

William Betz, Rochester, N.Y., 1927–1929, 1930–1931, 1934–36

Walter F. Downey, Boston, Mass., 1928–1929

Edwin W. Schreiber, Ann Arbor, Mich., 1928–1929

Elizabeth Dice, Dallas, Tex., 1928, 1929–1931

J. O. Hassler, Norman, Okla., 1928, 1929–1931, 1933

* Deceased

John R. Clark, New York City, 1929–1931

Mary S. Sabin, Denver, Colo., 1929–1930, 1931–1933

J. A. Foberg, California, Pa., 1929.

C. Louis Thiele, Detroit, Mich., 1931–1933

John P. Everett, Kalamazoo, Mich. 1932–1934

Elsie P. Johnson, Oak Park, Ill., 1932–1934

Raleigh Schorling, Ann Arbor, Mich. 1932–1934

W. S. Schlauch, New York City, 1933–1935

H. C. Christofferson, Oxford, Ohio, 1934–1936

Edith Woolsey, Minneapolis, Minn., 1934–1936

Martha Hildebrandt, Maywood, Ill 1934–1935

Official Primary Ballot

The National Council of Teachers of Mathematics, Atlantic City Meeting, February 22, 1935

For Second Vice-President, 1935-37 (Vote for two)

J AMISON, G. H.
Kirksville, M .

KEARNEY, DORA E.
Cedar Falls, Iowa

( ) MILLER, FLORENCE BROOKS
Shaker Heights, Ohio

( ) STOKES, C. N.
Philadelphia, Pa

( ) WALSH, SARA C.
Buffalo, N Y

For Members of the Board of Directors, 1935-38 (Vote for Six)

) ALENIUS, ALFHILD
Denver, Colo.

) BENZ, H. E.
Athens, Ohio

( ) BURROUGHS, FRED N.
Cleveland, Ohio

( ) FAWCETT, HAROLD P.
Columbus, Ohio

(HARTUNG, M. L.
Madison, Wis.

KENNEDY, KATHRYN
Terre Haute, Ind.

) LANE, RUTH
Iowa City, Iowa

LOSS, NELLIE
Flint, Mich.

McCUNE, MRS. SARA S.
New Castle, Pa.

) POTTER, MARY
Racine, Wis.

( ) ROCHE, NANETTE
Baltimore, Md.

( ) SMITH, ROLLAND R.
Springfield, Mass.

) TAYLOR, S. HELEN
Urbana, Ill.

WALLIS, WILLIAM J.
Washington, D. C.

WHITECRAFT, S. H.
Muncie, Ind

Members will please mark this ballot at once and mail same to Edwin W. Schreiber, Secretary, 425 E. Calhoun St., Macomb, Ill. Kindly put your name and address on the outside of the envelope. If you prefer to make a copy of this ballot on a separate sheet of paper it will be accepted.